Smith **Math 290** **Spring 2010**

Exam 1

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Define all of the following

D.1. [5 points] The **span** of the set $S = \{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n\}$.

D.2. [5 points] A **dependent variable** of the consistent system of linear equations $LS\left(A, \vec{b}\right)$.

D.3. [5 points] The **sum** of $\vec{u}$ and $\vec{v}$ where $\vec{u},\ \vec{v} \in \mathbf{C}^m$.

Do both of these "Computational" problems

C.1. [15 points] By hand, put the following matrix into reduced row-echelon form.

$$\begin{bmatrix} 1 & 1 & 2 & -1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{bmatrix}$$

C.2. [15 points] Suppose you have a system of n linear equations in k variables. Explain why It follows that

1. (a) both b.and c. below are correct
   (b) if the system has a unique solution, then $n \geq k$
      i. Unique solution means every column is a pivot column so must have enough equations for variables: $n \geq k$
   (c) if $n \geq k$, then the system has a unique solution
   (d) neither b. nor c. is necessarily correct.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Given the matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$ with column vectors $\vec{A}_1,\ \vec{A}_2,\ \vec{A}_3,\ \vec{A}_4$ and vector $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in \mathbf{C}^3$, prove that $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$ solves $LS\left(A, \vec{b}\right)$ if and only if $c_1\vec{A}_1 + c_2\vec{A}_2 + c_3\vec{A}_3 + c_4\vec{A}_4 = \vec{b}$. [You may **not** use In-Class Theorem 2.]

M.2. [15 points] [15 points] Suppose that $\vec{w}_1,\ \ \vec{w}_2 \in \mathbf{C}^m$. Prove that $\langle\{\vec{w}_1, \vec{w}_2\}\rangle = \langle\{\vec{w}_1, \vec{w}_2, 3\vec{w}_1 - 2\vec{w}_2\}\rangle$

M.3. [15 points] Suppose we run the sequence of three row operations: $\left[A \mid \vec{b}\right] \xrightarrow{R_2 \leftrightarrow R_3} [B \mid \vec{c}] \xrightarrow{\quad 3R_4 + R_1 \quad} \left[C \mid d\right.$ where

$$[D \mid \vec{e}] = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

What is $\vec{b}$?

Do any two (2) of these "Other" problems

T.1. [5, 10 points] Suppose that S is a set containing exactly two distinct vectors from $\mathbf{C}^m$.

1. (a) Prove that the sets S and $\langle S \rangle$ are not equal.
   (b) If $\vec{x},\ \vec{y} \in \langle S \rangle$. Prove that $\vec{x} + \vec{y} \in \langle S \rangle$.

T.2. [15 points] Let A be any $m \times 4$ matrix whose third column vector is the sum of its first two column vectors. Let B be the matrix that results from performing the row operation $\alpha R_i + R_j$ on A. Prove that the third column vector of B is the sum of the first two column vectors of B.

T.3. [15 points] Suppose A is an $n \times n$ matrix in which column vector $\vec{A}_j$ is twice column vector $\vec{A}_i$. Prove that A is singular.